# Multi-class to Binary reduction of Large-scale classification Problems

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# Outline

- Motivation
- Learning objective and reduction strategy
- Experimental results
- Conclusion

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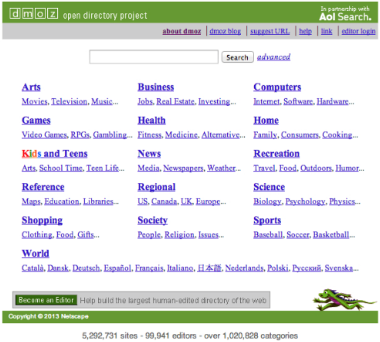
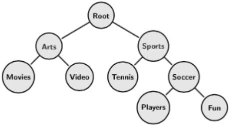
# Multiclass classification : emerging problems

- The number of classes, $K$, in new emerging multiclass problems, for example in text and image classification, may reach $10^5$ to $10^6$ categories.

- For example

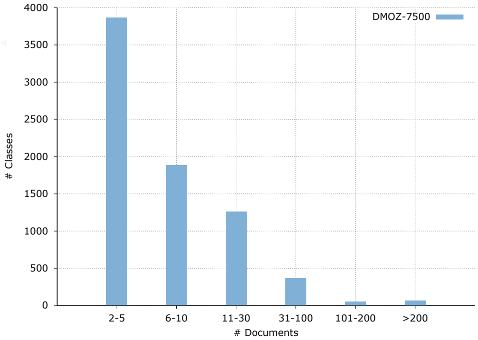



- $5 \times 10^6$ sites
- $10^6$ categories
- $10^5$ editors
- imbalanced nature of hierarchies
- Arbitrariness in taxonomy creation - personal biases



# Large-scale classification : power law distribution of classes

| Collection | $K$ | $d$ |
|---|---|---|
| DMOZ | 7500 | 594158 |



# Multiclass classification approaches

- Uncombined approaches, i.e. MSVM or MLP. The number of parameters, $M$, is at least $O(K \times d)$.
- Combined approaches based on binary classification :
  - One-Vs-one - $M \geq O(K^2 \times d)$
  - One-Vs-Rest - $M \geq O(K \times d)$
- For $K >> 1$ and $d >> 1$ traditional approaches do not pass the scale.

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## Learning objective

- Large-scale multiclass classification,
  - Hypothesis : Observations $\mathbf{x}^y = (x, y) \in \mathcal{X} \times \mathcal{Y}$ are i.i.d with respect to a distribution $\mathcal{D}$,
  - For a class of $\mathcal{H} = \{h : \mathcal{X} \times \mathcal{Y} \to \mathbb{R}\}$, a ranking instanstaneous loss $h \in \mathcal{H}$ over an example $\mathbf{x}^y$ by :

$$e(h, \mathbf{x}^y) = \frac{1}{K-1} \sum_{y' \in \mathcal{Y} \setminus \{y\}} \mathbb{1}_{h(\mathbf{x}^y) \leq h(\mathbf{x}^{y'})},$$

  - The aim is to find a function $h \in \mathcal{H}$ that minimizes the generalization error $L(h)$ :

$$L(h) = \mathbb{E}_{\mathbf{x}^y \sim \mathcal{D}} \left[ e(h, \mathbf{x}^y) \right].$$

  - Empirical error of a function $h \in \mathcal{H}$ over a training set $\mathcal{S} = \left(\mathbf{x}_i^{y_i}\right)_{i=1}^m$ is

$$\hat{L}_m(h, \mathcal{S}) = \frac{1}{m} \sum_{i=1}^m e(h, \mathbf{x}_i^{y_i})$$

# Reduction strategy

- Consider the empirical loss

$$\hat{L}_m(h,\mathcal{S}) = \frac{1}{m(K-1)}\sum_{i=1}^{m}\sum_{y'\in\mathcal{Y}\setminus\{y_i\}}\mathbb{1}_{h(\mathbf{x}_i^{y_i})\leq h(\mathbf{x}_i^{y'})}$$

$$= \underbrace{\frac{1}{n}\sum_{i=1}^{n}\mathbb{1}_{\tilde{y}_i g(\mathbf{Z_i})\leq \mathbf{0}}}_{L_n^T(g,T(S))}$$

where $n = m(K-1)$, $Z_i$ is a pair of couples costituted by a couple of example and its class and the couple corresponding to the example and another class, $\tilde{y}_i = 1$ if the first couple in $Z_i$ is the true couple and $-1$ otherwise, and $g(\mathbf{x}^y,\mathbf{x}^{y'}) = h(\mathbf{x}^y) - h(\mathbf{x}^{y'})$.

# Reduction strategy for the class of linear functions

**Input:** Labeled training set $\mathcal{S} = (\mathbf{x}_i^{y_i})_{i=1}^m$ ;
A binary classifier $\mathcal{A}$ ;
**Initialize**
$T(S) \leftarrow \emptyset$ ;
**for** $i = 1..m$ **do**
  **for** $k = 1..K$ **do**
    **if** $y_i > k$ **then**
      $T(S) \leftarrow \{(\Phi(\mathbf{x}_i^{y_i}) - \Phi(\mathbf{x}_i^k), +1)\}$
    **end**
    **if** $y_i < k$ **then**
      $T(S) \leftarrow \{(\Phi(\mathbf{x}_i^k) - \Phi(\mathbf{x}_i^{y_i}), -1)\}$
    **end**
  **end**
**end**
Learn $\mathcal{A}$ on $T(S)$

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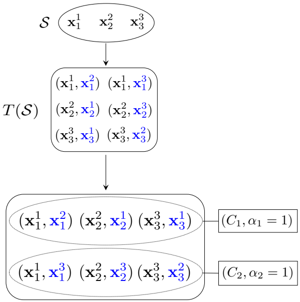
Learn $\mathcal{A}$ on $T(S)$

Problems :

- How to define $\Phi(\mathbf{x}^y)$,
- Consistency of the ERM principle with interdependant data.

# Consistency of the ERM principle with interdependant data

- Different statistical tools for extending concentration inequalities to the case of interdependent data,
- tools based on colorable graphs proposed by (Janson, 2004) [1].




1. S. Janson. Large deviations for sums of partly dependent random variables. Random Structures and Algorithms, 24(3) :234–248, 2004.

# Theorem

*Let* $\mathcal{S} = (\mathbf{x}_i^{y_i})_{i=1}^m \in (\mathcal{X} \times \mathcal{Y})^m$ *be a training set constituted of m examples generated i.i.d. with respect to a probability distribution* $\mathcal{D}$ *over* $\mathcal{X} \times \mathcal{Y}$ *and* $T(\mathcal{S}) = ((\boldsymbol{Z}_i, \tilde{y}_i))_{i=1}^n \in (\mathcal{Z} \times \{-1, 1\})^n$ *the transformed set obtained with application* $T$*. Let* $\kappa : \mathcal{Z} \to \mathbb{R}$ *by a PSD kernel, and* $\Phi : \mathcal{X} \times \mathcal{Y} \to \mathbb{H}$ *the associated mapping function. For all* $1 > \delta > 0$*, and all* $g_w \in \mathcal{G}_B = \{\mathbf{x} \mapsto \langle \boldsymbol{w}, \Phi(\mathbf{x}) \rangle \mid ||\boldsymbol{w}|| \leq B\}$ *with probability at least* $(1 - \delta)$ *over* $T(\mathcal{S})$ *we have then :*

$$L^T(g_w) \leq \epsilon_n^T(g_w, T(\mathcal{S})) + \frac{2B\mathfrak{G}(T(\mathcal{S}))}{m\sqrt{K-1}} + 3\sqrt{\frac{\ln(\frac{2}{\delta})}{2m}} \tag{1}$$

*where* $\epsilon_n^T(g_w, T(\mathcal{S})) = \frac{1}{n}\sum_{i=1}^{n} \mathcal{L}(\tilde{y}_i g_w(\boldsymbol{Z}_i))$ *with a surrogate Hinge loss* $\mathcal{L} : t \mapsto \min(1, \max(1 - t, 0))$*,* $L^T(g_w) = \mathbb{E}_{T(\mathcal{S})}[L_n^T(g_w, T(\mathcal{S}))]$ *et* $\mathfrak{G}(T(\mathcal{S})) = \sqrt{\sum_{i=1}^n d_\kappa(\boldsymbol{Z}_i)}$ *with*

$$d_\kappa(\mathbf{x}^y, \mathbf{x}^{y'}) = \kappa(\mathbf{x}^y, \mathbf{x}^y) + \kappa(\mathbf{x}^{y'}, \mathbf{x}^{y'}) - 2\kappa(\mathbf{x}^y, \mathbf{x}^{y'})$$

# Key Features of Algorithm

- Data dependent bound :
  If the feature representation of (x,y) pairs is independent of original dimension, then :
  $\mathfrak{G}(T(S)) \leq \sqrt{n \times Constant} \approx \sqrt{m \times (K-1) \times Constant}$
- Non-trivial joint feature representation (example-class pair)
- Same for any number of class
- Same parameter vector for all classes

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# Feature representation $\Phi(\mathbf{x}^y)$

| Features | |
|---|---|
| 1. $\sum_{t \in y \cap x} \ln(1 + y_t)$ | 2. $\sum_{t \in y \cap x} \ln(1 + \frac{l_{\mathcal{S}}}{\mathcal{S}_t})$ |
| 3. $\sum_{t \in y \cap x} I_t$ | 4. $\sum_{t \in y \cap x} \ln(1 + \frac{y_t}{\lvert y \rvert})$ |
| 5. $\sum_{t \in y \cap x} \ln(1 + \frac{y_t}{\lvert y \rvert}.I_t)$ | 6. $\sum_{t \in y \cap x} \ln(1 + \frac{y_t}{\lvert y \rvert}.\frac{l_{\mathcal{S}}}{\mathcal{S}_t})$ |
| 7. $\sum_{t \in y \cap x} 1$ | 8. $\sum_{t \in y \cap x} \frac{y_t}{\lvert y \rvert}.I_t$ |
| 9. $d_1(\mathbf{x}^y)$ | 10. $d_2(\mathbf{x}^y)$ |

- $x_t$ : number of occurrences of terme $t$ in document $x$,
- $\mathcal{V}$ : Number of distinct terms in $\mathcal{S}$,
- $y_t = \sum_{x \in y} x_t$, $|y| = \sum_{t \in \mathcal{V}} y_t$, $\mathcal{S}_t = \sum_{x \in \mathcal{S}} x_t$, $l_{\mathcal{S}} = \sum_{t \in \mathcal{V}} \mathcal{S}_t$.
- $I_t$ : idf of the terme $t$,

# Experimental results on text classification

| Collection | $K$ | $d$ | $m$ | Test size |
|---|---|---|---|---|
| DMOZ | 7500 | 594158 | 394756 | 104263 |
| WIKIPEDIA | 7500 | 346299 | 456886 | 81262 |

$$K \times d = O(10^9)$$

- Random samples of 100, 500, 1000, 3000, 5000 and 7500

# Experimental Setup

Implementation and comparison :

- SVM with linear kernel as binary classification algorithm
- Value of C chosen by cross-validation
- Comparison with OVA, OVO, M-SVM, LogT

Performance Evaluation :

- Accuracy : Correctly classified examples in test dataset
- Macro F-Measure : Harmonic mean of precision and recall

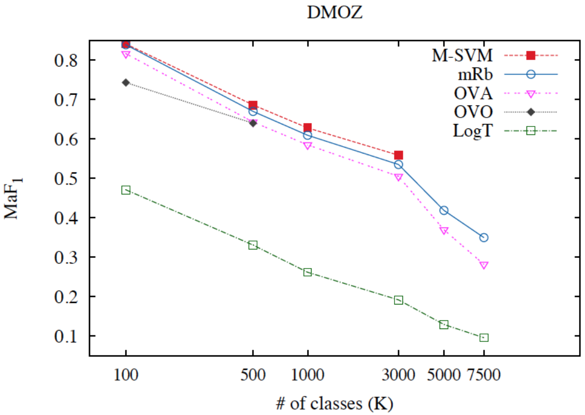
# Experimental Results

Result for 7500 class :

| | DMOZ-7500 | | | Wikipedia-7500 | | |
|---|---|---|---|---|---|---|
| | Acc. | MaF$_1$ | $N_c$ | Acc. | MaF$_1$ | $N_c$ |
| mRb | .479$^{\downarrow}$ | **.352** | .495 | .437$^{\downarrow}$ | **.378** | .551 |
| OVA | **.549** | .282$^{\downarrow}$ | .379 | **.484** | .348$^{\downarrow}$ | .489 |
| LogT | .311$^{\downarrow}$ | .096$^{\downarrow}$ | .194 | .231$^{\downarrow}$ | .151$^{\downarrow}$ | .287 |

- OVO and M-SVM did not pass the scale for 7500 classes
- $N_c$ : Proportion of classes for which at leaset one TP document found
- mRb covers 6-9.5% classes than OVA ( 500 - 700 classes)

# # of Classes Vs. Macro F-Measure



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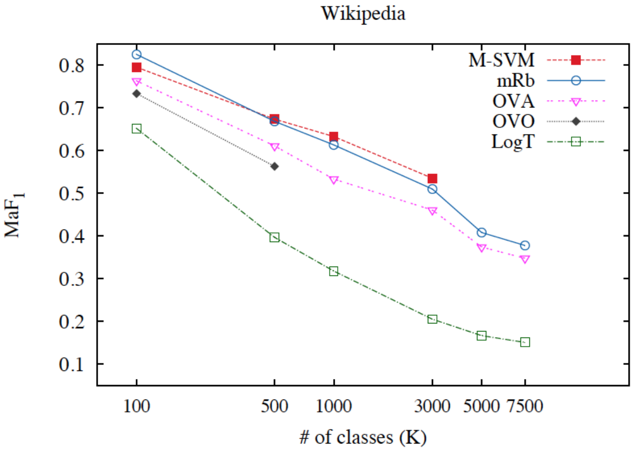

# Conclusion

- A new method of large-scale multiclass classification based on reduction of multiclass classification to binary classification.

- Efficiency of deduced algorithm comparable or better than the state of the art multiclass classification approaches.